



FORMULA SHEET · CLASSES 8 TO 12

IOQM and RMO Toolkit: Theorems, Formulas and Practice

Everything the Indian Mathematical Olympiad route leans on, from IOQM to RMO and INMO: the pattern of each paper, the theorems in number theory, algebra, combinatorics and geometry, and ten problems to try.

Maths Olympiad

01 The route and the papers

Stage	2026-27	The paper
IOQM	6 September 2026	Classes 8 to 12. 30 questions with answers from 00 to 99 in 3 hours: questions 1-10 carry 2 marks, 11-20 carry 3 and 21-30 carry 5. No negative marking
RMO	15 November 2026	For IOQM qualifiers: 6 problems with full written proofs, in 3 hours
INMO	17 January 2027	For RMO qualifiers: 6 proof problems in 4.5 hours
IMOTC	After INMO	A training camp at HBCSE, Mumbai, which picks India's team for the International Mathematical Olympiad

02 Number theory

Divisibility – by 3 or 9 if the digit sum is; by 11 if the alternating digit sum is; by 8 if the last three digits are

HCF and LCM – $\gcd(a, b) \times \text{lcm}(a, b) = ab$; $\gcd(a, b) = \gcd(b, a \bmod b)$

Number of divisors – $n = p^a q^b r^c$ has $(a+1)(b+1)(c+1)$ divisors

Sum of divisors – multiply $(p^{a+1} - 1)/(p - 1)$ over the prime powers

Fermat's little theorem – $a^{p-1} \equiv 1 \pmod{p}$ when p is prime and does not divide a

Euler's theorem – $a^{\phi(n)} \equiv 1 \pmod{n}$ when $\gcd(a, n) = 1$; $\phi(n) = n \times \prod (1 - 1/p)$

Wilson's theorem – $(p-1)! \equiv -1 \pmod{p}$ for every prime p

Zeros at the end of $n!$ – $n \div 5 + n \div 25 + n \div 125 + \dots$, keeping only whole parts

Squares – leave 0 or 1 when divided by 4, and 0, 1 or 4 when divided by 8

Chinese remainder theorem – remainders for coprime divisors fix the number modulo their product

Last digits – work modulo 10 or 100; powers repeat in short cycles

03 Algebra and inequalities

Quadratic $ax^2 + bx + c$ – roots add to $-b/a$ and multiply to c/a

Cubic $x^3 + px^2 + qx + r$ – roots add to $-p$, pairs multiply and add to q , all three multiply to $-r$

A sum of squares – $p^2 + q^2 = (p+q)^2 - 2pq$

Factorising – $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Sophie Germain – $a^4 + 4b^4 = (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$

AM $\geq$ GM – $(a+b)/2 \geq \sqrt{ab}$ for $a, b \geq 0$, with equality when $a = b$

Cauchy-Schwarz – $(a_1 b_1 + a_2 b_2 + \dots)^2 \leq (a_1^2 + a_2^2 + \dots)(b_1^2 + b_2^2 + \dots)$

Telescoping – $1/(k(k+1)) = 1/k - 1/(k+1)$

Sums – $1 + 2 + \dots + n = n(n+1)/2$; squares add to $n(n+1)(2n+1)/6$; cubes to $(n(n+1)/2)^2$

04 Counting

Choosing – $C(n, r) = n!/(r!(n-r)!)$; arranging r of n : $n!/(n-r)!$

Stars and bars – $x_1 + \dots + x_k = n$ in whole numbers ≥ 0 has $C(n+k-1, k-1)$ solutions

Inclusion-exclusion – $|A \cup B| = |A| + |B| - |A \cap B|$, and so on for three sets

Pigeonhole – $n + 1$ objects in n boxes put two in one box

Round a table – n different people sit in $(n - 1)!$ ways

Grid paths – from $(0, 0)$ to (m, n) , moving right or up, in $C(m + n, m)$ ways

Diagonals – a polygon with n sides has $n(n - 3)/2$

Catalan numbers – $C(2n, n)/(n + 1)$: 1, 1, 2, 5, 14, 42

05 Geometry

Angle bisector – the bisector from A cuts BC in the ratio $AB : AC$

Ceva – AD, BE and CF meet at one point exactly when $(BD/DC)(CE/EA)(AF/FB) = 1$

Power of a point – $PA \times PB = PC \times PD$ for two chords through P , and $= PT^2$ for a tangent PT

Cyclic quadrilateral – opposite angles add to 180° ; Ptolemy: $AC \times BD = AB \times CD + AD \times BC$

Area of a triangle – $\frac{1}{2}ab \sin C$; Heron: $\sqrt{s(s - a)(s - b)(s - c)}$; rs ; $abc/(4R)$

Sine rule – $a/\sin A = b/\sin B = c/\sin C = 2R$

Cosine rule – $c^2 = a^2 + b^2 - 2ab \cos C$

Centroid – divides each median in the ratio $2 : 1$

Right triangle – the circumcentre is the midpoint of the hypotenuse

06 How to work the IOQM paper

Small cases first – try $n = 1, 2, 3, 4$ and look for the pattern

Answers run from 00 to 99 – a huge answer usually means you missed a remainder or a digit sum

Pace – about 6 minutes a question: faster on 2-markers, slower on 5-markers

No negative marking – give an answer to every question, after checking

Old papers – solve past IOQM and PRMO papers in 3 hours, then study the solutions

07 Practice problems

1 mark each

- 1 Find the remainder when 2^{100} is divided by 7.

Your answer

- 2 How many positive divisors does 360 have?

Your answer

- 3 How many zeros does $125!$ end with?

Your answer

- 4 p and q are the roots of $x^2 - 7x + 3 = 0$. Find $p^2 + q^2$.

Your answer

- 5 Find the least value of $x + 16/x$ for $x > 0$.

Your answer

- 6 In how many ways can 10 identical sweets be shared among 3 children so that each child gets at least one?

Your answer

- 7 A right triangle has legs 20 and 21. Find its hypotenuse.

Your answer

- 8 How many diagonals does a 12-sided polygon have?

Your answer

- 9 Chords AB and CD of a circle meet at P inside the circle. $PA = 4$, $PB = 9$ and $PC = 6$. Find PD .

Your answer

- 10 Find the last two digits of 2^{20} .

Your answer

Answers and explanations

- 1 **02** $2^3 = 8 \equiv 1 \pmod{7}$ and $100 = 3 \times 33 + 1$, so $2^{100} \equiv 2$.
- 2 **24** $360 = 2^3 \times 3^2 \times 5$, so it has $(3 + 1)(2 + 1)(1 + 1) = 24$ divisors.
- 3 **31** Count the factors of 5: $125 \div 5 = 25$, $125 \div 25 = 5$ and $125 \div 125 = 1$, so $25 + 5 + 1 = 31$.
- 4 **43** $p + q = 7$ and $pq = 3$, so $p^2 + q^2 = (p + q)^2 - 2pq = 49 - 6 = 43$.
- 5 **08** By AM $\geq$ GM, $x + 16/x \geq 2\sqrt{16} = 8$, with equality at $x = 4$.
- 6 **36** Give each child one sweet first, then share the other 7 freely: $C(9, 2) = 36$.
- 7 **29** $20^2 + 21^2 = 400 + 441 = 841 = 29^2$.
- 8 **54** $n(n - 3)/2 = 12 \times 9 \div 2 = 54$.
- 9 **06** Power of a point: $PA \times PB = PC \times PD$, so $36 = 6 \times PD$ and $PD = 6$.
- 10 **76** $2^{10} = 1024$ ends in 24, and $24^2 = 576$ ends in 76, so 2^{20} ends in 76.

How to use this sheet

- 1 Check the dates and rules each year on the HBCSE Olympiad pages.
- 2 Write full solutions to every practice problem: RMO marks the proof, not the answer.
- 3 Keep a notebook of tricks you meet, sorted into the four topics.

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